The stretch response for all specimens was a nonlinear rise with increasing slope to a peak followed by relaxation (an exponential decline in tension). This was repeated in four initial trials. Four rates of stretch were applied, with a time to full displacement: 40, 100, 400 and 800 ms.

The resulting stretch-relaxation response was modeled using a state-variable representation. The equations for the tension force (F) are:

\[ z = A z + B x \]
\[ F = C z + D x \]

The constant matrices are A,B,C,D; z are internal states; x is input stretch. The parameters defining the model were obtained in two steps. First, the response over time, from the peak force to 21 seconds, was modeled from the exponential relaxation of an elastic component in parallel with three linear Maxwell (string-damper analogs) [1]. Second, the elastic, nonlinear component was the relaxed tension divided by the cube of the stretch input. This modeled the initial rise from zero tension to the peak. The resulting model then had four elements in parallel: a nonlinear elastic component and three time-dependent, linear Maxwell components.

RESULTS

The stretch response for all specimens was a nonlinear rise with increasing slope to a peak followed by relaxation (an exponential decline in tension). For stretch rates longer than 100 ms, the response generally had only two parts, an increase in force while the stretch was applied and smooth relaxation of force as the displacement was held. For the highest rate of stretch in most specimens, the relaxation response had an initial rapid decrease in tension following the peak force, with a more gradual decrease as stretch was maintained. The magnitude of the peak tension (stress) decreased as the rate of stretch became longer.

The state-variable representation of this response for each ligament was modeled using three time constants. These are \(0.081 \pm 0.019s, 1.053 \pm 0.271s, 10.753 \pm 1.824s\) for the ALL and \(0.085 \pm 0.022s, 0.971 \pm 0.262s, 11.628 \pm 1.628s\) for the PLL. The correlation of the model with experimental data was better than \(r^2 = 0.98\) for each ligament over the 21 seconds. When normalized for the magnitude of the relaxed stress, there was no significant difference between the response of ALL and PLL.

DISCUSSION

The objectives of this study were to derive models of the stretch-relaxation response of the cervical spine anterior and posterior longitudinal ligaments. Few studies have measured the effect of the rate of stretching on the viscoelastic responses of the human cervical spine [2], although rate-dependent effects have been shown [3]. This appears to be the first study to model the relaxation response of human cervical spine ligaments.

The use of state-variable to model the ligament response offered several advantages in comparison to common transform techniques. First of all the parameters needed to describe the unidirectional tension test could be obtained from the tension vs. time data beyond the peak load. The mean values of these constants provides a single canonical state-space model which appears to closely fits the ligaments response for the rates of ligament stretch and the range of displacements examined. Second, the experimental time-history data can be input directly to this single set of equations (with no transformation) to confirm the ligament response. The internal state of each component of the model is described throughout the solution. Sampling rate and aliasing common to discrete time models presented the only model limitations. Other limitations of the model include the effects of temperature, which were not considered here, and the possible interaction of tissue structures with the isolated components. The model provides a basis for describing more complete spine segments.

The model demonstrates several aspects of the ligament response. The model and the experimental data show the peak tension following stretch depends on the rate of stretch. Rapid manipulation can produce tension forces larger than 1.5 times forces produced at low rates. The relaxed tension however depends only on the amount of stretch and not on the rate. In relaxation studies of other ligaments (anterior cruciate ligament [1]), the peak force was constant and the final tension increased with repeated trials. The reason for this difference is not known.

The study showed that a single nonlinear elastic component and three internal viscoelastic states (parallel Maxwell components) are needed to accurately model the ligament relaxation. As found in lumbar ligaments, the internal damping of the ligament dissipates 27-39 percent of the mechanical energy applied during loading [4]. Here because the elastic response can be modeled simply as \(k_c x^3\), this suggests that more complicated nonlinear, quasi-static models must reflect contributions of time dependent components and are not elastic. The model and experimental data confirm that tension relaxes over the first 20-30 seconds following a manipulation.

An accurate description of the time dependent response of the cervical spinal ligaments is needed to refine spine modeling, surgical treatment and rehabilitation. The relaxation response of individual human cervical spine ligaments display significant differences in comparison to other ligaments.

REFERENCES


ACKNOWLEDGEMENTS

Funded by GM Research in agreement with the U.S. Department of Transportation.