**Modeling Variability and Uncertainty in the Experimental Response of Vertebral Trabecular Bone**

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**Introduction:** A major limitation of the application of computational methods in predicting the risk of fracture in skeletal structures is the inability to account for inherent variability and uncertainty in important system parameters such as biological tissue material properties, joint and muscle loading, and anatomy and the resulting effect of this variability on the computed model response. Experimental investigation has demonstrated that material behavior of human vertebral trabecular bone includes permanent (plastic) deformations and loss of stiffness due to damage accumulation, that plastic deformation and damage accumulation are time-dependent processes, and that material time-dependence changes with the evolution of damage [1]. The objective of the current work was to describe the variability and uncertainty in the nonlinear experimental stress response of cylindrical specimens of vertebral trabecular bone using a probabilistic computational framework.

**Materials and Methods:** A phenomenological constitutive model for trabecular bone was previously developed by assuming small strain theory and consolidating plasticity, damage mechanics, and viscoelasticity models within a unified stress-space constitutive model [2]. The plasticity model allows isotropic linear hardening and was developed as an orthotropic associated material model in stress space [3]. A nearly isotropic principal strain-based super-ellipsoidal surface was converted to a stress-based yield criterion and scaled for application to vertebral trabecular bone [4, 5]. An associated orthotropic damage accumulation model was developed assuming strain-energy equivalence, with a quadratic potential surface in damage space [6]. Uniform expansion of the damage surface is allowed using damage “strengthening” that is comparable to plastic hardening and is consistent with results for cortical bone suggesting that significant new damage does not occur until previous maximum strains are exceeded [7]. Viscous unloading behavior was modeled using a five-parameter Maxwell body expanded to an orthotropic model by treating the relaxation moduli as fourth-rank tensors [8]. Scalar parameters relating stiffness tensors for the viscous elements to the initial stiffness tensor were defined as functions of accumulated damage magnitude. In previous work, 10 cylindrical trabecular bone specimens were cored from thoracolumbar vertebrae with superior-inferior orientation and tested using a tensile loading protocol consisting of two trapezoidal pulses under tensile strain control [8, 9]. The DMG1 pulse had a peak strain of 0.8% and the DMG2 pulse had a peak strain of 1.2% (60 second hold periods, 180 second zero-strain recovery periods following unloading). The apparent level isotropic response of specimens in uniaxial loading was modeled and appropriate material parameters were determined for each specimen. Isotropic stiffness tensors were defined using experimental tangent modulus and assuming a Poisson’s ratio of 0.3. The orthotropic damage characteristic tensor [9] and yield criterion parameters [4] were determined in previous studies. The remaining material model parameters [viscoelastic parameters: two parameters relating the stiffness tensors and two time constants; plasticity parameters: yield criterion scaling parameter and isotropic plastic hardening modulus; and damage parameters: an energy threshold for damage onset and isotropic “damage strengthening” modulus] were determined by minimizing the RMS error between experimental and predicted axial stress over the loading history for the DMG1 trapezoidal pulse for each specimen (MATLAB, The Mathworks, Inc., Natick, MA). Lognormal random variable distributions were determined for all material parameters. The constitutive model and random variable distributions were implemented along with correlations between random variables using MATLAB within a probabilistic analysis framework (NESSUS, Southwest Research Institute, San Antonio, TX). Random variable distributions were sampled 1000 times using Latin hypercube probabilistic sampling and the probabilistic stress response to simulated strain loading for the DMG1 and DMG2 pulses was determined.

**Results:** The probabilistic implementation of the trabecular bone constitutive model successfully predicts the variability observed in the experimental stress-strain response for 10 vertebral trabecular bone specimens. The mean and variance of the stress response are well predicted for DMG1 and, aside from an initial stress offset, the shape of the general loading response is predicted for DMG2. Damage is underestimated during the loading ramp of the DMG2 pulse.

**Discussion:** The history dependence of the mechanical behavior of vertebral trabecular bone provides a challenging set of problems, even with the controlled geometry and loading of experimental investigations. Although this work represents a successful initial description of the variability in the inelastic effects of vertebral trabecular bone, a significant amount of work remains in identifying relationships between underlying micro- and ultrastructural mechanisms, in determining the suitability of such a modeling framework for more general loading conditions, and providing statistical support for such a model on the basis of additional response data. We adopt the view that modeling the process of damage accumulation and incorporating probabilistic descriptions of input parameter uncertainty and variability are critical to predictive success. The ability to model variability in the characteristic features of the highly nonlinear behavior of heterogeneous trabecular bone allows us to move somewhat closer to the goal of predicting the in vivo response or risk for a specific clinical patient based on noninvasive diagnostic measures of bone.

**References:**

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