INTRODUCTION There are high empirical correlations between strain rate, cortical and cancellous bone apparent stiffness (Young's modulus), apparent yield strength, apparent ultimate strength and cortical bone fracture toughness [1, 2, 3]. A mechanistic explanation for these correlations is lacking.

Microcracking is a major mechanism in cortical and cancellous bone failure, however, microdamage content alone cannot explain the strain rate dependence of bone strength without considering viscoelastic energy dissipation mechanisms of the crack. Using a viscoelastic micromechanical model of a fiber bridged microcrack and data from the literature, we demonstrate that the experimental apparent yield strength of bone [2] can be predicted directly from measurements of apparent moduli of elasticity of bone constituents and failure strain of the collagenous matrix.

METHODS THE MODEL Our interest is to predict apparent bone tissue strength from the apparent stiffness of bone constituents and the failure strain of bone matrix with the fundamental assumption that bridged microcracking is the major mechanism in determining yield in bone (Fig. 1). Fiber bridging is a means of toughening a material where the tendency of small cracks to propagate is reduced by the presence of fibers that interconnect the faces of the crack. For a single crack under remote tension and with arbitrary crack closure stress in an infinitely large medium, the crack opening displacement as a function of the other parameters of the model is [5]:

$$\delta(X) = \frac{4a}{\pi E} \int_{X}^{X'} \frac{s - \sigma_0 - \delta(X)}{\sqrt{X^2 - X'^2}} \, ds$$

(Eq. 1)

where $s$ and $t$ are integration variables, $X=x/a$ is the distance along the crack normalized by the half-crack-length $a$. $\sigma_0-\delta(X)$ is the stress on the crack faces (calculated from the fiber stress $\sigma_0$ and the fiber volume fraction $\nu_0$) that resists the opening of the crack. $\delta(X)$ is our case the yield strength of the bone, $\delta$ the crack opening displacement, $E$ the elastic modulus of the bone and $\nu$ its Poisson's ratio. The elastic modulus of the collagenous fibers and the bone are assigned the form $E_f = E_0 f_j$ and $E = E_0 \nu$ [2, 6], where $E$ and $\nu$ are the apparent strain rate in bone and the strain rate in the bridging fiber, respectively. $E_0$, $E_f$, $m$ and $n$ are constants.

Fiber stresses will be related to maximum crack opening displacement ($\delta_0$) and a characteristic fiber length, $L_c$, that represents the length of bridging fiber that participates in creating the closing stress. The fiber is a linkage between points beneath the opposing crack faces (Fig. 1). The crack closure stresses are assumed to be linearly distributed to obtain:

$$p(X) = V_f \sigma_f = V_f E_0 \frac{\delta_0}{L_c} \left(1 - X \frac{\delta_0}{L_c}\right)^m \left(1 - X_m\right)^n$$

(Eq. 2)

where $\delta_0 / L_c$ and $\delta_0 / L_c$ are the maximum fiber strain and strain rate, respectively. First, equation 2 was linearized on $X$ using Taylor series expansion and then was solved together with eqn 1 using MATHCAD for the remote (apparent) stress $\sigma_0$:

$$\sigma_0 = \frac{E_0}{4n} \frac{\nu_0 E_f V_f \nu f_m}{\nu_0 E_f V_f \nu f_m}$$

(Eq. 3)

THE INPUT DATA All of the parameters in Eq. 3 can be estimated from published data. For example, the apparent remote stress $\sigma_0$, stiffness ($E_f$) and Poisson’s ratio ($\nu$) were estimated from data for the human femur as 108-117 MPa (yield stress) [7], 15600-18300 MPa [7], and 0.3, respectively, obtained at a strain rate of 0.02-0.05 s⁻¹ [7]. The range of the bridging fiber strain $\epsilon_0$ was estimated as 6.3-13.7% (failure strain) from the data for demineralized bone measured at a strain rate of 0.003 s⁻¹ [8]. A toe-in strain (the initial non-linear strain observed in tensile testing of demineralized bone) of 2.05% was subtracted from this range to better represent the fiber strains in situ [9]. The apparent fiber stiffness ($E_f \nu f$) was estimated as 171-475MPa [8]. The power constants $m$ and $n$ were set as 0.06 [2, 6]. The crack width ($2a$) was estimated as 50 micrometers [10]. $E_0$ and $E_f$ were calculated from $E$ and $E_f$ and the strain rate at which $E_0$ and $E_f$ were measured.

First, a range of $L_c$ was determined using Eq. 3 while $\epsilon_0$ and $\nu_0$ had the values at which the bone properties were measured. Then using the mid-point value of this range for $L_c$, the strain rate dependency of the yield strength was determined.

RESULTS For the estimated parameter range, the absolute maximum and minimum estimated characteristic lengths were 12.7 micrometer and 3.6 micrometer, respectively (Fig. 1). For the midvalue of $L_c$, 6 µm, using eqns 1 and 2, it was predicted that the contribution of the bridged microcrack to strength increases with increasing strain rate, the predicted values of yield stress being within 10% of the values determined by the empirical relationship reported by Currey [2] for a mineralization of 64% for human femur (Fig. 2).

DISCUSSION Using literature data for the stiffness of human cortical bone constituents, we were able to predict the dependence of apparent strength on strain rate using a mechanistic model with an accuracy better than 90%. The micromechanical model presented here recreates the empirical relationship between apparent stiffness and strength [2] without the need to model higher level cortical structures such as Haversian systems. This suggests that for cortical bone the empirical relationship between apparent stiffness and strength could be a property of the hard tissue and flaws in it rather than a result of bone's intermediate architecture.

Our characteristic length prediction (3-13 µm) is consistent with collagen fibrils [11] crossing between the crack faces and a part of their length being stretched during crack opening. If crack bridging and reinforcement occurs at the fibrillar level (consistent with the characteristic length prediction of our model), any condition or treatment that changes the bonding of one fibril to another or to the mineralized matrix will change the apparent strength of the tissue. For example, decreased collagen crosslinking reduces bone strength (lathyrism) and relaxin or NKISK can affect collagen fiber sliding [12].


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