Predicting Whole Metatarsal Fatigue Life with Finite Element Modeling

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INTRODUCTION: Bone fatigue is a silent threat that can lead to fractures or bone stress injuries. In older adults, it manifests as fragility fractures from falls at standing height or lower. In athletes, bone stress injuries often occur first, impeding continued participation in recreational, sports, or military activities. Sub-maximal but post-yield loads cause strain hardening, making bone stiffer and more brittle, thus reducing its overall toughness. The concept of strained volume, which measures the accumulated volume of a part that exceeds yielding strains, has been shown to be a practical and efficient predictor of fatigue life in materials such as bone[1]. While fatigue damage can signal the need for bone remodeling—a crucial process for maintaining bone health—excessive use can lead to microdamage accumulation, significantly increasing the risk of injury. This study focuses on the metatarsals, a common site for bone stress injury, leveraging the similarity in shape and size among metatarsals 2, 3, and 4 for more comprehensive data collection. Predicting a bone's fatigue life—its loading cycles to failure—is challenging due to the complex, variable composition of bone at both micro- and macro-scale levels. A reliable method to predict bone fatigue life would enhance our understanding of bone toughness and improve strategies for injury prevention and treatment. Although the Finite Element Method (FEM) has been widely studied to predict bone fatigue life, most research focuses on small specimens and animal models, reducing their relevance to *in-vivo* human bone mechanics[2,3]. This study aims to address these limitations by applying FEMs to whole human bone samples loaded under physiologically relevant conditions *ex-vivo*. Furthermore, we will challenge the use of isotropic material properties to solve for von Mises effective strains by employing anisotropic material properties combined with the Tsai-Wu multi-axial failure criteria. We hypothesize that this approach will improve the accuracy of predicting metatarsal stiffness loss compared to traditional von Mises criteria.

METHODS: A total of 27 metatarsals were dissected from 14 cadaveric human feet. Each bone was imaged with High-Resolution peripheral Quantitative Computed Tomography (HRpQCT, 246 µm voxel size, Xtreme CT I, Scanco) to quantify the shape and hydroxyapatite mineral density distribution. The metatarsals were fixed proximally in impact-resistant polyurethane resin. Each metatarsal was loaded either axially or at 30^{o} with a force-controlled 4 Hz sine wave load applied to the distal end, up to 250,000 cycles. The target force was chosen to be static and within a range that would produce stress associated with bone failure within 250,000 cycles, determined by a threshold of maximum von Mises strain between 3000-5000 µe. Fatigue life was quantified by matching the number of loading cycles to 10%, 20%, and 30% stiffness loss. For the FEMs, the CT images of the metatarsals were segmented and processed (Mimics 26.0, 3-Matic 18.0) to produce ten-node tetrahedral meshes with adaptive element sizes that have a maximum edge length of two millimeters. Linear-elastic material properties were assigned based on a density-modulus optimization method proposed by Fung[4] and previously established orthotropic relationships. Elements within ten millimeters of the proximal end of each metatarsal were fully constrained, while nodes within ten millimeters of the distal end were kinematically coupled to avoid artificial strain concentrations. Static simulations were created to replicate the metatarsal-specific mechanical testing conditions, using the target force. Data from elements located five or more millimeters away from constrained nodes were recorded for each simulation. Strained volume was calculated by applying Tsai-Wu or von Mises failure criteria on element strains to normalize for the varying elastic moduli. The volumes were then regressed with the fatigue life metrics in a log-log S-N curve, and performance was evaluated by the t-statistic of the model slope as well as the coefficients of determination (R^2), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE).

RESULTS: The von Mises strained volume models failed to reach statistical significance, while the Tsai-Wu estimated strained volume models consistently reached significance ($p < 0.05$) with approximately 20% of the fatigue life variance explained (Table 1). Therefore, we accept the central hypothesis that Tsai-Wu damage criteria improve human metatarsal stiffness loss accuracy compared to traditional von Mises criteria.

Table 1. Comparison of Stiffness Loss Prediction Accuracy Using von Mises and Tsai-Wu Strained Volume Criteria.

Stiffness Loss	Strained Volume Criterion	R^2	p-value	RMSE (cycles)	MAE (cycles)
10%	von Mises	0.07	0.191	766.26	531.87
10%	Tsai Wu	0.23	**0.011**	759.92	548.02
20%	von Mises	0.05	0.275	771.53	542.95
20%	Tsai Wu	0.21	**0.017**	770.79	557.33
30%	von Mises	0.04	0.296	770.05	546.19
30%	Tsai Wu	0.18	**0.025**	771.43	555.12

DISCUSSION: This study aimed to highlight the limitations of and improve FEMs designed to predict bone fatigue damage. We compared two similar FEMs, both with anisotropic, density-dependent material properties. While von Mises failure criteria enforce a directionally independent maximum strain state, the Tsai-Wu criteria consider asymmetric failure thresholds between tension, compression, and shear. The results show that Tsai-Wu criteria more effectively represent bone damage across single and multi-axial loading scenarios.

SIGNIFICANCE/CLINICAL RELEVANCE: Accurate prediction of bone fatigue damage is crucial for preventing stress injuries and fractures. This study demonstrates that the Tsai-Wu failure criteria provide a more reliable estimate of metatarsal stiffness loss compared to traditional von Mises criteria. Improved prediction methods can enhance strategies for injury prevention and treatment, offering better insights into bone health management, especially in athletes and older adults at risk of fractures.

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